The Generalized Discrete Cost Multicommodity Network Design Problem with Stochastic Demands

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Keywords

Network Design, Network Flows, Stochastic Optimization.

Problem Presentation

In this work, we address the Stochastic Discrete Cost Multicommodity Network Design Problem (SGDCMNDP). The input data include a connected undirected graph $G= (V, E)$, where V is a set of n nodes and E is a set of m edges, and a set of $K=n(n-1)/2$ distinct point-to-point commodities. Each commodity k is characterized by a stochastic demand flow d_k to be routed along several paths (bifurcated routing) between a specified source node s_k and a specified sink node t_k (k=1,…,K). A per-unit penalty α_k is associated to each unrouted demand unit of commodity k. Moreover, a set of L_e discrete facilities/modules is available and can be installed on each edge $e \in E$. These facilities may correspond, for instance, to roads or highways in transportation networks, to transmission facilities in telecommunication networks, to medium or high voltage electric lines in energy systems, etc. Each facility l, $l=1... L_e$, is characterized by a bidirectional capacity u_l^e and a fixed cost f_l^e that are discrete step-increasing functions (i.e., $u_1^e<u_2^e<...<u_{Le}^e$, and $f_1^e<f_2^e<...<f_{Le}^e$). The SDCMNDP requires installing at most one facility on each edge in order to minimize the sum of the fixed facility installation costs and the expected penalty of unrouted demands.

Related work

Network Design Problems (NDP) represent a central class of combinatorial optimization problems that arise in a wide range of fields such as telecommunications, distributed computer networks, transportation and logistics, and energy systems. A comprehensive survey on NDPs' taxonomy was introduced by Magnanti and Wong since 1984 [1]. In the literature, several variants of stochastic NDPs have been studied by OR researchers, mainly single-facility stochastic NDPs (e.g. [2]) and single-commodity stochastic NDPs (e.g. [3]). To the best of our knowledge, this is the first work that investigates a stochastic network design problem with both multi-commodity and multi-facility features.

Solution Approach

In order to solve the SDCMNDP, we developed a simulation-optimization approach to determine the network configuration that minimizes the total expected costs, i.e. the fixed installation costs and the penalties for the unrouted demands. The proposed approach is applied to a path-based formulation that considers a hedging flow parameter. It is noteworthy that the proposed path-based model requires the use of a column generation algorithm to derive Benders cuts. We assume that demands are stochastic and carry significant uncertainty. To tackle this uncertainty, we propose a two-level procedure. At the first level, the facilities to be installed are determined via an exact cut generation approach. Then, the routing problem is solved by a Monte Carlo simulation based routine. Iteratively, the hedging quantities are updated heuristically.

Furthermore, we solved a sequence of Sample Average Approximation problems in order to derive a valid lower bound to the SDCMNDP that provides valuable information about the quality of the approximate solutions.

Computational Results

To assess the performance of the proposed approach, we conducted extensive computational experiments on randomly generated instances. We used C# language in concert with the MILP solver CPLEX 12.5 to implement all the proposed procedures. Computational results illustrate the effectiveness of the proposed approach in generating high-quality solutions to instances with up to 30 nodes in a reasonable computation time.

References

[1] Magnanti, T. L., & Wong, R. T. (1984). Network design and transportation planning: Models and algorithms. *Transportation science*, *18*(1), 1-55.

[2] Rahmaniani, R., Crainic, T. G., Gendreau, M., & Rei, W. (2018). Accelerating the Benders Decomposition Method: Application to Stochastic Network Design Problems. *SIAM Journal on Optimization*, *28*(1), 875-903.

[3] Shen, S., You, M., & Ma, Y. (2017). Single-commodity stochastic network design under demand and topological uncertainties with insufficient data. *Naval Research Logistics*, *64*(2), 154-173.