Non-efficient points approach for solving multiobjective linear problems

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Several works have been devoted to the task of finding an appropriate solution method for a multiobjective linear problem (MOLP). An MOLP can be written as:

$$Max\{C.x: x \in X\}$$

with

$$X = \{x \in \mathcal{R}^n / \, Ax \leq b, x \geq 0\}$$

where C is the objective matrix of order $l * n$ so that $C^k, k = 1, \dots, L$, is the k^{th} objective function of our problem. A is a constraint matrix of order $m * n$ and b is a vector of order $m * 1$ corresponding to the right-hand side values of the constraints.

The solution for the MOLP is the set of all non-dominated solutions called the Pareto (or efficient) set. We note that:

- A solution x is said to be feasible only if $x \in X$.
- $x^1 \in X$ is dominated by $x^2 \in X$ if $Cx^1 \in Cx^2$. (Iserman, 1977)
- A feasible point x^0 is said to be efficient with respect to X and C, if there is no $x' \in X$ such that $Cx^0 \leq Cx'$ (Iserman, 1977). Hence, a dominated point is a non-efficient point.

To generate all efficient solutions of an MOLP Yu and Zeleny (1975) proposed a solution method consists in finding the initial efficient vertex, then enumerating the set of all efficient vertices and finally, generating the set of all efficient faces. The two last steps are two algorithms having an interactive structure where the output of the first is the input of the second.

In 1980, Ecker et al. presented a new algorithm to deal with MOLP problems based on generating all efficient vertices. For each new efficient vertex, they list all maximal efficient faces incident to that vertex. This approach was improved by Armand (1993) who provides a recursive structure algorithm, with an advantage in terms of the time execution of the corresponding implemented program.

In this work and in opposition to all the above methods, we define a new approach for solving MOLP problems relying on non-efficient vertices named Non-efficient points approach (NEPA). Then, we exploit NEPA to implement a robust algorithm which, broadly, generates solutions in a brief time. This performance is noticeable for large MOLP problems.

Keywords: Linear programming; Multiobjective Linear Programming; Efficient Solution

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