The SISINA Method: A Distance-Based Multi-Attribute Ranking Approach with Superiority and Inferiority Indexes

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Abstract The SIR-TOPSIS method proposed by Xu in 2001 used either the superiority indexes or the inferiority ones to rank multi-attribute alternatives. Unlike the SIR-TOPSIS, the SISINA (**S**uperiority **I**deal-**S**eeking and **I**nferiority **N**adir-**A**voiding) method developed in this contribution exploited simultaneously both indexes. Another major difference is that to compute the proximity value of the superiority profile of each alternative to the superiority ideal (i.e., the goal) the weighted Manhattan distance metric was used, and to compute the remoteness value of its inferiority profile from the inferiority anti-ideal (i.e., the nadir) the weighted Chebyshev distance metric was used. The desirability value of each alternative is derived easily from the previous values. A real world multi-attribute ranking problem is provided with a view to illustrating the feasibility of the suggested method.

Keywords: Multi-attribute ranking, Inferiority index, SIR-TOPSIS, Superiority index.

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1. Introduction

In 2001, Xu [4] used the notions of *superiority* and *inferiority scores* presented in [2, 3] along with the notion of *generalized criterion* defined in [1] to come up with the so-called superiority and inferiority *indexes* which gave rise, *inter alia*, to the superiority and inferiority ranking (SIR) method variant called SIR-TOPSIS. The SIR-TOPSIS method used either the superiority indexes or the inferiority ones to rank multi-attribute alternatives. In order to make full use of the available information, the current contribution proposes a distance-based multi-attribute ranking approach named the *SISINA* (**S**uperiority **I**deal-**S**eeking and **I**nferiority **N**adir-**A**voiding) method which uses *simultaneously* the superiority and inferiority indexes of the predefined alternatives. Further, knowing that given two points x, y $\in \Re^n$, where $\Re$ is the set of real numbers, the Chebyshev distance between them is shorter than or equal to their Manhattan distance. For this reason, the SISINA method uses these two different distance metrics: the weighted Manhattan distance metric to compute the proximity value of each

superiority profile to the superiority ideal (i.e., the goal) and the weighted Chebyshev distance metric to compute the remoteness value of each inferiority profile from the inferiority nadir.
In this regard, the SISINA method could be judged as a "cautious" approach to the ranking of multi-attribute alternatives.

In this contribution, we will provide the necessary definitions and notation, and the various steps of the orderly ranking process of the SISINA method, and treat a real world case study on logistics service provider selection.

2. Definitions and notation

In this section, we will start by giving the problem description and some definitions and notation.

2.1 Problem description

The formal description of the problem at hand can be stated as follows.
Given:

- m pre-specified alternatives $\{A_i\}_{i=1}^{i=m}$;
- n benefit and/or cost, ordinal and/or cardinal evaluation attributes $\{g_j\}_{j=1}^{j=n}$;
- An attribute weight vector $W = (w_1, w_2, \dots, w_n)$ satisfying $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$;
- A $m \times n$ decision matrix, $[a_{ij}]$, where a_{ij} denotes the evaluation of alternative A_i with respect to attribute g_j $(for\ i = 1\ to\ m\ and\ j = 1\ to\ n)$.

Goal:

To rank the various pre-specified alternatives using the SISINA method.

2.2 Essential concepts

Hereafter, we will define the superiority index (S-index) $S_j(A_i)$ and the inferiority index (I-index) $I_j(A_i)$ of any alternative A_i with respect to attribute g_j as in [ ]. The S-index and I-index of any alternative A_i $(for\ i = 1\ to\ m)$ with respect to attribute g_j $(for\ j = 1\ to\ n)$ are respectively defined by:

- $$S_j(A_i) = \sum_{K=1}^{m} P_j(A_i\ , A_K) = \sum_{K=1}^{m} f_j(a_{ij} - a_{kj}) = \sum_{k=1}^{m} f_j(d_{ik}) \qquad (1)$$

- $I_j(A_i) = \sum_{K=1}^{m} P_j(A_K, A_i) = \sum_{K=1}^{m} f_j(a_{kj} - a_{ij}) = \sum_{k=1}^{m} f_j(d_{ki})$ (2)

The S-indexes are benefit indicators (i.e., more is better) whereas the I-indexes are cost indicators (i.e., less is better), in addition they belong to the closed real interval [0, m - 1].

In what follows each alternative A_i $(for\ i = 1\ to\ m)$ will be characterized by the following two multi-attribute descriptive profiles:

$$S_i = (S_1(A_i), S_2(A_i), ..., S_n(A_i)) \quad \text{(The superiority profile of } A_i\text{)} \tag{3}$$

$$I_i = (I_1(A_i), I_2(A_i), ..., I_n(A_i)) \quad \text{(The inferiority profile of } A_i\text{)} \tag{4}$$

The superiority ideal $\boldsymbol{S^+}$ and the inferiority nadir $\boldsymbol{I^-}$ are respectively defined as follows:

$$S^+ = (\max_{1\le i\le m} S_1(A_i), \max_{1\le i\le m} S_2(A_i), \dots, \max_{1\le i\le m} S_n(A_i)) \quad \text{(Ideal)} \tag{5}$$

$$I^- = (\max_{1\le i\le m} I_1(A_i), \max_{1\le i\le m} I_2(A_i), \dots, \max_{1\le i\le m} I_n(A_i)) \quad \text{(Nadir)} \tag{6}$$

The i*th* proximity value π_i to the superiority ideal is defined by:

$$\pi_i = d_1(S_i, S^+) = \sum_{j=1}^{n} w_j (\max_{1\le k\le m} S_j(A_k) - S_j(A_i)) \tag{7}$$

where d_1 is the weighted Manhattan distance metric, whereas, the i*th* remoteness value ρ_i from the inferiority nadir is given by:

$$\rho_i = d_\infty(I_i, I^-) = \max_{1\le j\le n}(w_j (\max_{1\le k\le m} I_j(A_k) - I_j(A_i)) \tag{8}$$

where d_∞ is the weighted Chebyshev distance metric.

Then, the desirability value δ_i of each alternative is founded as:

$$\delta_i = \frac{\rho_i}{\rho_i + \pi_i} = \frac{d_\infty(I_i, I^-)}{d_\infty(I_i, I^-) + d_1(S_i, S^+)}, \quad i = 1 \text{ to } m \tag{9}$$

2.3 Orderly ranking process

The different steps of the orderly ranking process of the SISINA method with precise numerical importance weights are set out below.

Step 1. Form the superiority and inferiority profiles.

Step 2. Determine the superiority ideal S^+.

Step 3. Determine the inferiority nadir I^{-}.

Step 4. Compute the proximity values π_i, i = 1 to m.

Step 5. Compute the remoteness values ρ_i, i = 1 to m.

Step 6. Compute the desirability values δ_i , i = 1 to m.

Step 7. Rank the alternatives in descending order of their desirability values.

3. Conclusion

Unlike the SIR-TOPSIS method, the SISINA method developed in this work uses simultaneously the so-called inferiority and superiority indexes of the predefined alternatives. It is founded on seeking the superiority ideal (i.e., approach behavior) and avoiding the inferiority nadir (i.e., avoidance behavior). The proximity and remoteness values of each alternative are computed differently by using respectively the weighted Manhattan and the weighted Chebyshev distance metrics. For this reason the SISINA method could be seen as "Cautious" compared to other distance-based multi-attribute ranking methods.

References

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