European option pricing by simulation based on the retrograde stochastic differential equations

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Abstract:

The retrograde stochastic differential equations noted as RSDE are considered as a recent theory in full swing as they are developing without regard through their applications in various fields among which we note: finance and stochastic control. These were introduced for the first time in 1973 in an article by J.M. Bismut in the linear case then we had wait until the early 90's and the work of E. Pardoux and S. Peng to have the first result for the non-linear case. Since then, many works have been done and the theory has continued to develop due to its close relationship with financial mathematics and stochastic differential equations noted as SDE.

The RSDE are involved in many modeling problems, especially in finance for the valuation of derivatives, so it appears important to have flexible and effective algorithms to approach them. Indeed, the main purpose of this article is to solve a common problem in finance that is the pricing of options. For example, a European call, with maturity T and strike price K is a contract which gives the right but not the obligation to its holder to buy a share of the share at the strike price K on the date T. The seller of the option therefore undertakes to pay the holder the amount $(S_T - K)^+$ which represents the profit that the exercise of the option allows. More generally, one can imagine a contingent asset whose profit is a positive random variable ξ which depends on $(S_t; 0 \leq t \leq T)$. So, our problem is to determine at which price should we sell the option and that the seller must ensure that by selling the option at this price on the date $t = 0$, he will have the

sum ξ at the date t = T. In this context, we therefore aim to solve the stochastic differential equations retrograde.

In this paper, we have three chapters: The first will deal with some notions of stochastic calculus as well as stochastic differential equations. The second will revolve around the retrograde stochastic differential equations and we will show the classical result of existence and uniqueness. The third one will include the different numerical simulations for the SDE and the RSDE and in which we will present an example of application for the calculation of the selling price of a European call option through an algorithm implemented in Python to finally compare the result obtained with the price of the option observed on the market in order to highlight the advantage of the application of probabilistic methods such as the RSDE which provide more accurate and realistic results than the deterministic methods.

Key-words: *retrograde stochastic differential equations, simulation, stochastic calculus, European option.*