Medical equipment maintenance optimization: Mixed-Integer Linear Programming
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Abstract: The main target of the maintenance department in the hospital is to guarantee the patient safety by properly keeping up the medical devices. Any potential hazard due to the bad performance of the devices can have severe consequences on the patient life. In this paper, we propose a Mixed Integer Linear Programming (MILP) model to: 1) select the best maintenance strategy, e.g., run-to-failure, time-based, and conditional based, for each equipment in the hospital, 2) decide on the best option for insourcing or outsourcing maintenance activities per equipment, 3) optimize the tactical maintenance decisions per equipment. Maintenance service in the hospital has limited resources to maintain the medical devices. Therefore, by selecting which equipment to be maintained in-house or to be outsourced and the contract to be used for outsourcing are considered as tactical decisions. The objective is to minimize the total annual maintenance costs without affecting the availability of critical devices.

Keywords: Mixed Integer Linear Programming, Medical devices, Maintenance strategies.

1. INTRODUCTION

Nowadays, modern medical equipment are becoming more and more complex and sophisticated. To insure reliability and safety of medical devices, a number of support services are made to properly manage these devices in a hospital. Maintenance service is one of these services. The World Health Organization (WHO, 2011) refers to the maintenance plan as a way to execute a maintenance strategy that is able to keep medical equipment in a healthcare institution reliable, safe and available for use.

Preventive maintenance (PM) of medical devices is carried out either at fixed time intervals (Time-based maintenance) or when a given condition variable crosses a pre-specified limit (Condition-based maintenance). The aim of the preventive maintenance interventions is to decrease the risks of devices becoming non-operational and to obtain the greatest availability as possible. Khalaf (2004) state that the greatest problem, in developing countries, is not the lack of medical devices but there are more than 50% of devices are non-operational. Therefore, it is necessary to organize the maintenance activities by ensuring the equipment proper functioning (e.g., availability, reliability, and patient satisfaction), and preserving the cost efficiency of maintenance activities and the effective use of resources (staffing and tools).

According to Masmoudi et al. (2014, 2016), there are three major service and support possibilities for maintaining the medical equipment: in-house biomedical maintenance service, Original Equipment Manufacturers (OEM), and independent third-party service provider (with or without contracts). Based on the equipment criticality and the available budget of the maintenance department, the decision makers determine the sourcing decisions of maintenance activities: in-house or outsourced with or without a service contract.

For hospitals, the types of contracts differ from one country to another (Georgin et al., 2005). In the Tunisian context, Letaief et al. (2007) identify the following four possible contracts:

- Contract type A: All tasks of time-based maintenance (TBM) are performed by the subcontractor with labor and spare parts costs included in the maintenance package.
- Contract type B: All tasks of TBM and/or condition-based maintenance (CBM) are performed by the subcontractor with only spare parts included in the package.
• Contract type C: For this contract, there is no package included. In general, it is used for the corrective maintenance tasks when failures are complex.

• Contract type A*: All maintenance operations: corrective and preventive (TBM and CBM) are completely performed by the subcontractor with labor and spare parts costs included in the package.

Our goal in this paper is to provide a practical solution to hospitals that helps in determining: the maintenance strategy per equipment, the insourcing-outsourcing decisions per equipment, and the contract type in case of outsourcing. Moreover, we do also tactical decisions on time based and condition based for each medical device based on a tradeoff between costs and quality. This is using by a time efficient Mixed Integer Linear Problem (MILP) model.

2. PROBLEM DESCRIPTION

The selection of maintenance strategies and insourcing/outourcing decisions is a challenging problem, especially, for cases with a limited available maintenance budget planning and skills resource.

2.1 Model assumptions

In the model, we discretize the planning time horizon into intervals with an equal size, e.g., an interval is a month. In addition, we consider a planning time of one year.

The model assumptions are as follow:

• Corrective maintenance actions are considered as ‘minimal repairs’. The equipment returns to the same state just before failure. This means, after a repair the equipment is not "as good as new". Note, this assumption is realistic, especially, for complex equipment consisting of many components which its failure can be attributed to failure of just very few of them. By replacing these failed components in a corrective maintenance action brings the equipment to a state that was almost just before failure.

• Preventive maintenance action is scheduled, if any, in the beginning of month \( m \) (if we have planned PM for the month \( m \)). The state of the equipment after a preventive maintenance is "as good as new".

• The age of the equipment has no effect on the failure rate function. This is because during operations an equipment may receive several preventive maintenance actions which brings back to as good as new state.

• At most one preventive maintenance action can be done in a month for an equipment. However, there can be multiple minimal repairs per equipment per period.

• We assume that the medical devices have a failure rate that is proportional in time.

2.2 Proposed model

**Minimize** \( D \) (1)

\[
B - \sum_{i=1}^{N} \sum_{s=1}^{S} \sum_{j=1}^{6} G_{i,s,j} \leq D \tag{2}
\]

\[
C_{i,s,j} \cdot (1 - X_{i,s,j}) \cdot B \leq G_{i,s,j} \leq C_{i,s,j} \forall i = 1...N, s = 1...3, j = 1...6 \tag{3}
\]

\[
0 \leq G_{i,s,j} \leq B \cdot X_{i,s,j} \forall i = 1...N, s = 1...3, j = 1...6 \tag{4}
\]

\[
\sum_{s=1}^{S} \sum_{j=1}^{6} X_{i,s,j} = 1 \forall i = 1...N \tag{5}
\]

\[
\sum_{s=1}^{S} Z_{i,s} = 1 \forall i = 1...N \tag{6}
\]

\[
Z_{i,1} + 2Z_{i,2} + 4Z_{i,3} \leq Z_{i+1,1} + 2Z_{i+1,2} + 4Z_{i+1,3} \forall i = 1...N - 1 \tag{7}
\]

\[
\sum_{i=1}^{N-1} C_{r,i} (Z_{i,1} - Z_{i+1,1}) = T_1 \tag{8}
\]

\[
\sum_{i=1}^{N-1} C_{r,i} (Z_{i,1} - Z_{i+1,1} + Z_{i,2} - Z_{i+1,2}) = T_2 \tag{9}
\]
\[ C_{i,s,j} = CF_{i,s,j} + C_{4,s,j}(NP_{i,s}R_{j,s} + NC_{i,s}L_{j,s}) + C_{SP_{i,s,j}}(NP_{i,s}R_{j,s} + NC_{i,s}U_{j,s}) \quad i=1...N, s=1...3, j=1...6 \quad (10) \]

\[ X_{i,s,1} \leq Ho_{i,s} \quad \forall i=1...N, s=1...3 \quad (11) \]

\[ NC_{i,s} = \sum_{m=1}^{12} \int_{\eta_m}^{\beta_m} \lambda_i,m(t) \, dt \quad \forall i=1...N, s=1...3 \quad (12) \]

\[ NP_{i,s} = \sum_{m=1}^{12} l_{i,s,m} \quad \forall i=1...N \quad (13) \]

\[ l_{i,s,m} \leq Z_{i,s} \quad \forall i=1...N, s=1...3, m=1...12 \quad (14) \]

\[ TE_{i,m} + W_{i,m} \leq A_{i,m} \quad \forall i=1...N; m=1...12 \quad (15) \]

\[ \sum_{i=1}^{N} W_{i,m} \leq C_a_m \quad \forall m=1...12 \quad (16) \]

\[ W_{i,m} = TIP_{i,m} \cdot (l_{i,3,m} + l_{i,2,m}) + TIC_{i,m} \]

\[ \cdot \int_{\eta_m}^{\beta_m} \lambda_i(t) \, dt \quad \forall i=1...N, m=1...12 \quad (17) \]

\[ \lambda_i(\eta_m) = r_{oi} \cdot l_{i,3,m} + r_{cai} \cdot l_{i,2,m} + \lambda_i(\eta_{m-1}) \cdot e^{b_i(\eta_m-\eta_{m-1})} \cdot (1 - l_{i,2,m} - l_{i,3,m}) \quad \forall i=1...N, m=1...12 \quad (18) \]

\[ 0 \leq \lambda_i(\eta_m) \leq (l_{i,3,m} \lambda_{\max_i} + l_{i,2,m} \lambda_{cb i}) \quad \forall i=1...N, m=1...12 \quad (19) \]

\[ 1 \leq T_1 < T_2 \leq 5.12 \quad (20) \]

\[ D; B; N; \lambda_{\max_i}; \lambda_{cb i}; TIP_{i,m}; TIC_{i,m}; W_{i,m}; C_a_m; A_{i,m} \geq 0 \quad (21) \]

\[ K_{0,s,j}, R_{s,j}, L_{s,j}, U_{s,j}, H_{0,s,j}, X_{i,s,j}, l_{i,s,j}, Z_{i,s} \in \{0,1\} \quad (22) \]

The objective function (1) minimizes the deviation \( D \) between the total cost and the available Budget. Note, the cost of preventative maintenance is more expensive than of corrective maintenance (minimal repair) but more beneficial for the equipment. Therefore, the objective function will guarantee that more equipment are maintained preventively which leads to better operations of equipment. Moreover, our objective will guarantee how to allocate the fixed budget while respecting the availability level of equipment in each month \( A_{i,m} \).

### 2.5 Model linearization

The constraints (12), (17) and (18) are non-linear. In order to assure the linearity, let us first introduce

\[ f_i = \int_{\eta_m}^{\beta_m} e^{b_i(t-\eta_m)} \, dt \cdot e^{(b_i(\beta_m-\eta_m)-1)} \quad \text{Note, } \beta_m - \eta_m \text{ is a constant independent of } i \text{ and } m. \] Then, we linearize the non-linear constraints.

### 3. NUMERICAL RESULTS

In order to evaluate the behavior of our proposed model in a real case we present here the numerical results. The MILP model is solved by “CPLEX 15.2” and ran on a computer with a processor of the following characteristics: Intel (R) Pentium (R) CPU 2020M 2.60 GHz. We consider real input data of the medical devices based on the ‘Habib Bourguiba SFAX’ hospital in Tunisia. We tested our MILP model with many instances varying between 10 and 500 devices. Table 1 shows the computation time in seconds for these instances.

<table>
<thead>
<tr>
<th>No of instance</th>
<th>Number of medical devices</th>
<th>Computation time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>2.10</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>7.68</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>14.97</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>18.48</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>27.16</td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>73.54</td>
</tr>
</tbody>
</table>
In Table 2, we show the results of the selected maintenance strategies for 100 critical medical devices, with criticality thresholds $T_1 = 1.28$ and $T_2 = 1.3$. These results show a high efficiency computation time even for large problem size. We also tested the same instance (100 medical devices) by reducing the available budget by 5%.

Table 3 shows that when the maintenance budget is reduced (5%), we have more medical devices with corrective maintenance and less ones with preventive maintenance with criticality thresholds $T_1 = 1.3$ and $T_2 = 1.56$. For insourcing/outsourcing decisions, Table 3 presents a comparison of the number of contracts and equipment maintenance insourcing when the budget is reduced by 5%. In fact, by reducing the available budget, some equipment maintenance are maintained in-house and in general, less contracts are performed.

### Table 2. Numerical Results of MILP for 100 medical equipment (Maintenance strategies selection)

<table>
<thead>
<tr>
<th>Equipment</th>
<th>Corrective maintenance</th>
<th>Condition based maintenance</th>
<th>Time based maintenance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equipment 1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Equipment 19</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Equipment 20</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Equipment 23</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Equipment 24</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Equipment 100</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 3. Comparative Results of MILP for 100 medical equipment (Maintenance mode selection): insourcing/outsourcing and contract type.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>With available budget</td>
<td>0</td>
<td>1</td>
<td>27</td>
<td>5</td>
<td>4</td>
<td>63</td>
</tr>
<tr>
<td>With reducing 5% of avail. budget</td>
<td>1</td>
<td>3</td>
<td>15</td>
<td>14</td>
<td>13</td>
<td>54</td>
</tr>
</tbody>
</table>

### 4. CONCLUSIONS

The new contribution of this paper is to help the decision makers to allocate maximum medical devices as possible to preventive maintenance actions and internal biomedical maintenance service to reduce the total maintenance cost and increase the equipment availability by maximizing preventive maintenance activities. To realize this, a mixed integer linear programming MILP is proposed. This model allows selecting the best maintenance strategy and its mode for each medical device according to equipment’s criticality and the available budget.

### REFERENCES


