Non-efficient points approach for solving multiobjective linear problems

Fouad Ben Abdelaziz¹, Houda Alaya², Tarak Joulak³

¹NEOMA Business School, Rouen Campus Boulevard André Siegfried - 76130 Mont-Saint-Aignan, France Fouad.BEN.ABDELAZIZ@neoma-bs.fr

²Tunis Business School, Tunisia and Logistics & innovation technology research center, IHE paris <u>Houdaalaya@gmail.com</u>

> ³Logistics & Innovation technology research center, IHE paris <u>Tarak.joulak@gmail.com</u>

Several works have been devoted to the task of finding an appropriate solution method for a multiobjective linear problem (MOLP). An MOLP can be written as:

$$Max\{C. x: x \in X\}$$

with

$$X = \{x \in \mathcal{R}^n / Ax \le b, x \ge 0\}$$

where *C* is the objective matrix of order l * n so that C^k , k = 1, ..., L, is the k^{th} objective function of our problem. *A* is a constraint matrix of order m * n and *b* is a vector of order m * 1 corresponding to the right-hand side values of the constraints.

The solution for the MOLP is the set of all non-dominated solutions called the Pareto (or efficient) set. We note that:

- A solution x is said to be feasible only if $x \in X$.
- $x^1 \in X$ is dominated by $x^2 \in X$ if $Cx^1 \in Cx^2$. (Iserman, 1977)
- A feasible point x^0 is said to be efficient with respect to X and C, if there is no $x' \in X$ such that $Cx^0 \leq Cx'$ (Iserman, 1977). Hence, a dominated point is a non-efficient point.

To generate all efficient solutions of an MOLP Yu and Zeleny (1975) proposed a solution method consists in finding the initial efficient vertex, then enumerating the set of all efficient vertices and finally, generating the set of all efficient faces. The two last steps are two algorithms having an interactive structure where the output of the first is the input of the second.

In 1980, Ecker et al. presented a new algorithm to deal with MOLP problems based on generating all efficient vertices. For each new efficient vertex, they list all maximal efficient faces incident to that vertex. This approach was improved by Armand (1993) who provides a recursive structure algorithm, with an advantage in terms of the time execution of the corresponding implemented program.

In this work and in opposition to all the above methods, we define a new approach for solving MOLP problems relying on non-efficient vertices named Non-efficient points approach (NEPA). Then, we exploit NEPA to implement a robust algorithm which, broadly, generates solutions in a brief time. This performance is noticeable for large MOLP problems.

Keywords: Linear programming; Multiobjective Linear Programming; Efficient Solution

References

H. Iserman (1977) "The enumeration of the set of all efficient solutions for a linear multiple objective program" Opl Res. Q. 28. 3, 711 - 725.

P.L.Yu and M. Zeleny (1975) "The set of all non-dominated solutions in linear cases and a multicriteria simplex method" J. Math. Anal. App. 49, 430 – 468.

J.G. Ecker, N. S. Hegner and I.A. Kouada (1980) "Generating all maximal efficient faces for multiple objective linear programs" J.Opt.Theo. Application 3à, 353 – 381.

P. Armand (1993) "Finding all efficient faces in multiobjective linear programming" Mathematical Programming 61, 357 – 375.