

# A stochastic model for the design of electric vehicle charging infrastructure

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**Abstract.** We study the problem of designing electric vehicle (EV) charging infrastructure. The main decision that we consider is the location of the charging stations in a way that EV drivers can drive along the road network without running out of charge. We take into account the uncertainty of the driving range, which is the maximum distance that a fully charged EV can travel before its battery runs empty. We thus propose a stochastic model and maximize the expected coverage of the recharging demand. We first formulate a new mixed-integer linear program (MILP) for the stochastic problem and compare it with a previously published one. We then develop a Tabu search heuristic method to solve large-size instances of the problem. We carry out our numerical experiments using randomly generated road networks and we show the performance of the new formulation as well as the quality of the solutions provided by the Tabu search heuristic approach.

**Keywords:** Flow refueling location model, electric vehicle charging station network design, stochastic range.

## 1. Introduction

The problem of optimizing the design of an EV charging infrastructure is mostly related to a well-known problem in the Operations Research field, which is the facility location problem. Basically, facility location models aim at finding the best possible locations for facilities in order to optimize a performance criterion such as cost minimization or demand coverage maximization. Reviews on the academic literature on facility location can be found e.g. in [1]. In this paper, we study a facility location model for the design of an EV charging infrastructure. The aim of this model is to determine the best locations for EV charging stations in a way to maximize the coverage of the charging demand. Different approaches have been used in the literature to model this type of demand. The first approach assumes that the demand is located at a set of fixed points in space, i.e. is expressed at the nodes of the underlying network. An example of such models can be found in [2]. These models implicitly consider that drivers will carry out a special-purpose round trip from their home or workplace to the charging station in order to recharge the battery. However, in some cases, drivers do not specifically travel to the station to recharge their vehicles but rather recharge it while on their way to another destination. For EVs, this occurs in particular during long-distance trips exceeding the vehicle range. In this case, demand should be represented as a set of origin-destination paths rather than a set of nodes. Such a representation was first proposed by Hodgson [3] within his flow capturing location model. The main issue in this model is that it does not take into account the limited range of EVs, i.e. the maximum distance that a fully charged EV can travel before its battery runs empty. This was the motivation behind the development of the flow refueling location model by Kuby and Lim in

[4]. In this model, a flow on a given path is considered as refueled if and only if there is an adequate number of stations on the path, allowing an EV driver to refuel his journey from the origin to the destination. Different extensions of this model have been proposed in the literature in order to study more realistic assumptions such as e.g. the capacity of charging stations [5] or the deviations from the shortest paths [6]. However, little attention has been given to studying the uncertainties on the input data of the optimization problem, in particular the driving range uncertainty. To the best of our knowledge, the uncertainty of the driving range has been introduced in only two recent works. The first work [7] uses a probabilistic approach to model the range uncertainty and the second work [8] introduces two stochastic models, one maximizing the expected EV flow covered and the other one is a chance constrained model. In this paper, we present a new formulation for the problem of locating EV charging stations under uncertainties on the driving range, with the objective of maximizing the expected flow coverage.

## 2. Problem description and modeling

We consider a road network  $G(N,A)$ , where  $N$  denotes a set of nodes and  $A$  denotes a set of arcs linking these nodes. The charging demand to be satisfied by the stations is modeled as a set of flows denoted by  $Q$ . Each flow  $q \in Q$  is described by its origin  $O_q$ , its destination  $D_q$  and the number of EVs traveling along it  $f_q$ . Drivers belonging to flow  $q$  are assumed to follow the shortest path between  $O_q$  and  $D_q$ . All EVs have a limited range  $R$ , which is subject to uncertainty due to e.g. the traffic conditions, the weather or the age of the battery. A flow  $q$  is said to be covered if there is an adequate number of stations on this flow that allows EVs to travel from  $O_q$  to  $D_q$  without running out of charge. In other words, the flow  $q$  is covered when the distance between each pair  $k,l$  of consecutive stations on the flow does not exceed the range  $R$ , otherwise, it is not covered. In order to model the problem that determines the best locations for a predetermined number of stations while maximizing the expected flow covered on the network, we use three types of decision variables. Variable  $x_j$  is a binary variable equal to 1 if a station is opened at node  $j$ , 0 otherwise. Variable  $z_q \in [0,1]$  is a continuous variable equal to the probability of covering flow  $q$ . Finally, variable  $w_{kl}^q$  is a binary variable equal to 1 if a vehicle traveling along flow  $q$  is recharged at a station located at node  $k$  to cover the vehicle trip up to node  $l$  and 0 otherwise. The objective function of our MILP model can be written as follows: maximize  $\sum_{q \in Q} f_q z_q$ , i.e. maximizing the sum of EV flows on the network, weighted by their probability of coverage. We include in the model different sets of constraints that link variables  $z_q$  and variables  $w_{kl}^q$ , define the relationship between variables  $w$  and variables  $x$  and limit the number of charging stations that must be opened to a predetermined number  $p$  representing the limited investment budget. We assume that for a given realization of the random conditions  $w$ , the value of the range  $R(w)$  is the same for the whole network and that  $R(w)$  is randomly distributed following the cumulative density function  $G: R \rightarrow [0,1]$ . By using these assumptions and the fact that  $G$  is a non-decreasing function, we can write the range constraints as follows:

$$z_q \leq 1 - \sum_{k \in N_{lq}^b} G\left(t_q(k,l)\right) w_{kl}^q \quad \forall q \in Q, l \in N_q \setminus O_q, \text{ where } t_q(k,l) \text{ is the total length of arcs visited when traveling from node } k \text{ to node } l \text{ on flow } q \text{ and } N_{lq}^b \text{ is the set of nodes situated along flow } q \text{ before node } l \text{ on a trip from } O_q \text{ to } D_q.$$

### 3. Tabu search heuristic approach

For large size instances of the stochastic EV charging station location problem, the computation time required to get an optimal solution might become prohibitively long. Therefore, we propose a Tabu search procedure in order to obtain good quality solutions in short computation times. The algorithm starts by building an initial solution and setting the best feasible solution to the initial solution. Then, two different steps are alternated in the iterations of the algorithm. Step1 consists of selecting a station to be opened among the  $N-p$  closed stations. The station should not be Tabu (not recently closed) and should lead to the highest expected coverage among all possible openings. Step2 consists of selecting a station to be closed among the  $p + 1$  opened stations. This station should not be Tabu (not recently opened) and should lead to the highest expected coverage among all possible closings. The Tabu search procedure stops when the number of iterations without improvement of the best objective reaches a maximum limit.

### 4. Main conclusions from the numerical experiments

In our numerical experiments, we compare the performance of the new formulation proposed in this paper with the one of the previously published formulation in [7] for the expected flow refueling location model. We also analyze the quality of the solutions provided by the Tabu search heuristic approach. The road networks that we use in the tests were randomly generated following a procedure similar to the one proposed in [8]. We consider two different instances sizes: ( $N=100$ ,  $Q=1225$ ) and ( $N=200$ ,  $Q=4950$ ) and different values for the number of stations  $p$ . The random range  $R$  is represented using a Gamma distribution, with a shape parameter of 50 and a scale parameter of 5. For each instance size and each value of  $p$ , we generate 5 instances and consider the average output value. We employed the C++ language to implement the model and the commercial solver CPLEX 12.6.2 to solve it. All tests were carried out on a PC with Intel i5-3210M Core 2 Duo (2.50 GHz) with 8GB of RAM. The CPU time of CPLEX solver was limited to 10 hours. Examples of results are shown in Table 1. The results show that our formulation performs better than the existing one as it leads to significantly decreasing the CPU time. Moreover, when using the existing formulation, CPLEX failed at finding the optimal solution within the time limit for several instances. In those cases, the optimality gap went up to 13%.

**Table 1.** Average performance of the three methods (5 replications).

Instance	New formulation CPU (s)	Existing formulation CPU (s)	TABU CPU (s)	TABU Gap (%)
<i>N100Q1225p10</i>	165	583	9	0.2
<i>N100Q1225p25</i>	609	11008	24	0.1
<i>N200Q4950p10</i>	8212	32775	58	0.8
<i>N200Q4950p25</i>	26385	36000	184	0.6

The last column of Table 1 reports the gap between the solution found by the Tabu Search heuristic and the optimal solution. It shows that the heuristic performs very well as it provides good quality solutions in short computation times.

## 5. Conclusion

In this paper, we studied the problem of EV charging infrastructure planning under range uncertainty. We proposed a new MILP formulation for the problem, explicitly including the driving range. Then we developed a Tabu search heuristic approach to solve large-size instances. Our numerical experiments showed the performance of the new formulation as well as the quality of the solutions provided by the heuristic method. However, in our model, we assumed that for any realization of the random conditions, the value of the driving range is the same for all flows. An interesting direction for further research would thus be to relax this assumption and to study the more realistic case where the driving range realization is different on each cycle segment of a flow.

## 6. References

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