Monopoly pricing in an $M/M/1$ unreliable queue with strategic customers

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Abstract

In this paper, we study the service pricing and the strategic behavior of customers in an unreliable and totally unobservable $M/M/1$ queue, under a rewards-cost structure. We model the system using the two-step dynamic game theory tools, in order to analyze the decision process and determine explicitly the equilibrium strategies for the decision-makers (server-customers), according to their objectives.

Key Words: Strategic customers, Pricing, Queueing, Unreliable Server, Two-step dynamic game, Nash Equilibrium, Optimization.

I. Introduction

The literature on the strategic behavior of customers in queuing systems is very rich. This literature has given many useful ideas in the conception of queueing systems with rational customers, whose analysis combines two different and important disciplines namely: queueing and game theory.
The study of the rational queueing systems was initiated by Naor [13], who studied an $M/M/1$ queue with a reward-cost structure, where the system state is observable. His work was completed by Edelson and Hildebrand [5] in the unobservable case. Since then, there has been an increasing number of papers that deal with the strategic behavior of customers in different queuing systems. The manuscripts of Hassin and Haviv [7] and Hassin [8] present the main approaches and results on the subject.

In some of these works, authors seek to determine the optimal price of service to be offered by the system according to the strategic behavior of the customers. Chen and Frank [2] have adapted the classic Naor's model [13], where the server (firm) can adjust his service price according to the current state of the queue. Then, Chen and Frank [3] completed this work in the unobservable case.

The works cited so far deal with reliable queueing systems. However, in many real-world situations, the server is subject to random failures that affect the system characteristics. Among the works that have dealt with the analysis of the strategic behavior of the customers in queueing models with unreliable server, we can cite that of Economou and Kanta [6] which extended the model of Naor 13, including the failures and the repairs periods of the server while considering the totally observable case and the almost observable case.
More later, Li et al [11] completed this work by dealing with two other types of information: the almost unobservable case and the totally unobservable case. As for Wang and Zhang [14], they generalized the model studied in [6] by considering an $M/M/1$ queuing system with an unreliable server and delayed repairs. Yu et al. [16] completed this model in the almost unobservable case and then in the totally unobservable case.
Li et al. [10] analyzed the customers behavior in a Markovian server queue with partial breakdowns where, when a failure occurs, customers are served with a low rate compared to the service rate when the

system is operational, by treating two cases: totally observable and totally unobservable. Boudali and Economou [1] have studied the behavior of customers who decide whether to join or not a Markovian queuing system with catastrophes by analyzing two cases: the totally observable and the totally unobservable. Chen and Zhou [4] studied the customers behaviors in a Markovian queue with setup time and breakdowns, where three cases were considered: totally observable, almost unobservable and totally unobservable.

Most of the works mentioned so far consider a game among customers. Recently, another type of modeling has emerged in the queuing systems analysis with reward-cost structure. The latter is represented by dynamic game between the different agents (customers, server, social optimizer, ...) intervening in the system. Ziani et al. [17] studied an $M/M/1$ queue with three agents (the social optimizer, the service provider, and the customers), where they analyzed the interactions that occur between them and modeled the problem as a three-stage dynamic game. Manou et al. [12] studied a queueing model representing a transport station using a two-stage game between the customers and the administrator, where customer strategies depend on the level of information on the queue (totally observable, totally unobservable or almost observable). Jagannathan et al. [9] investigated a decision-making process between customers and two servers in a cognitive radio network, where a Markovian queueing system is considered with the server 2. The server 1 can adjust its service price first in order to maximize its revenue. Then, customers have the opportunity to buy a service from server 1 or enjoy a free service from server 2 that can be interrupted by random breakdowns. Upon their arrival, customers observe the price imposed by the server 1 and the state of the server 2, but they do not have information on the number of customers in the queue facing the server 2, then they decide between use server 1 or use server 2. Wang and Zhang [15] contributed to the analysis of a Markovian retrial queue and delayed vacations for Local Area Network applications in which the server will take delayed vacations at the end of the service. For LAN applications, they do not only studied the strategic behavior of customers but also considered the strategic monopoly (service provider). They modeled this situation as a two-stage game (Stackelberg competition) between the service provider and the customers.

As we mentioned above, there are many works that have considered the server breakdown problem and others that have considered the service pricing problem in the study of the Markovian queueing systems with strategic customers. Our work combines these both problematic. We study the pricing of the service provided the in an $M/M/1$ queueing system with an unreliable server and strategic customers where the system state is totally unobservable. We use the game theory tools to model the strategic interactions between the server and the customers and we analyze the impact of the service price selected by the server on customers' decisions and the consequences of the customer's decision on the utility of the server. To do that, we propose a two-stage game model between the server (leader) and the customers (followers). The resolution of the game determines the optimal service price and the equilibrium strategic decisions of the customers on their arrival to the system.

II. Model Description

We consider a Markovian $M/M/1$ queue with an unreliable server and an infinite capacity in which customers arrive according to a Poisson process with rate λ. The service times are assumed to be exponentially distributed random variables with rate μ and the service is performed according to FCFS policy. We assume that interarrival times and service times are mutually independent. The server is subject to random breakdowns following a Poisson process with parameter θ. When a failure occurs, the server becomes inactive then his repair is immediately started with an exponential distribution of rate r. This system is described by the process $\{N(t), I(t), t \geq 0\}$, where $N(t)$ represents the number of customers in the system and $I(t)$ represents the server state.

Furthermore, we assume that there is a reward-cost structure that reflects the desire of customers for the service to be acquired. A customer will pay a price P (service price) proposed by the server, receives a reward of R units (service value) if the service is performed and loses a waiting cost C per unit of time for the time remaining in the system. We also associate a fixed reward noted by v, which represents the reward of a customer who decides to choose an outside opportunity. This outside opportunity may reflect a customer's satisfaction without being served, it may also reflect a customer's gain if it is served elsewhere.

We assume that the server acts first and chooses a service price P that it will apply to all customers. Customers arrive sequentially to the system and they are informed of the price displayed by the server. They also have knowledge about the system parameters, but they are not informed on the system state (the exact number of customers in the queue and the server state). Each customer arriving to the system should choose between: join the system or choose an outside opportunity.

Therefore, each customer arriving to the system is at the same level of information as the customers who arrived before him and those who will arrive after him. Thus, only the service price P and the system parameters will be used for a customer's decision making. Given the homogeneity of customers, we can assume that a proportion of a q customers decides to join the system and another proportion $(1-q)$ customers chooses the outside opportunity.

Thus, the server interaction with the customers and the sequencing in the decision process bring back to a two-stage dynamic game. At the first stage of the game, the server imposes a service price P. At the second stage, even if the customers arrive to the system sequentially and each arriving customer must make his decision to enter or choose an outside opportunity on the basis of one and the same information for all, which is the price that the service displays, the appropriate game model would be a sequential game with imperfect information with an unknown number of players (customers). We can then consider that customers make their decisions simultaneously.

For a service price P and a proportion q of customers deciding to join, the server utility would be:

$$U_1(P, \alpha = (q, 1-q)) = \lambda q P \tag{1}$$

and the customer utility would be:

$$U_2(P, \alpha = (q, 1-q)) = q(R - P - C.\overline{T_s}) + (1-q)v. \tag{2}$$

where

$$\overline{T_s} = \frac{\mu\theta + (r+\theta)^2}{(r+\theta)(r\mu - \lambda q(r+\theta))}. \tag{3}$$

is the mean sejour time of a customer in the system.

In this paper, we show that the game thus constructed admits a perfect Nash equilibrium and we give explicitly the server equilibrium strategies and those of customers.

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